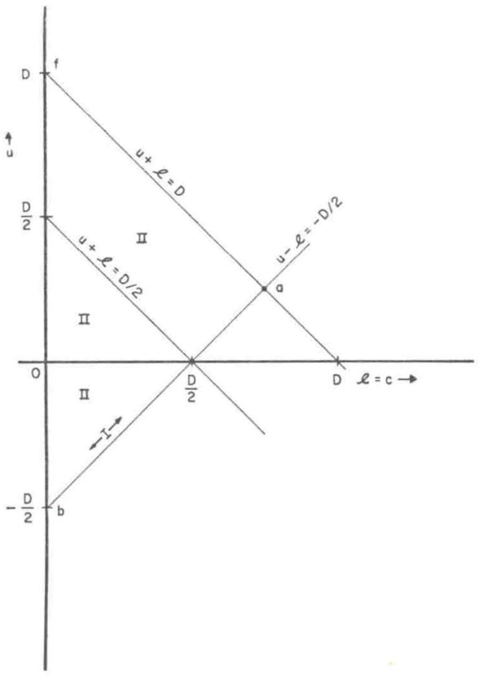

Figure 2
Hodograph plane for flow of Figure 1, case E1.

Region I in the (x, t) plane is bounded by the detonation front OP, by the C+ characteristic OF, and by the C− characteristic PG. The Chapman-Jouget state along OP is represented by point "a" in Figure 2, and the forward-facing rarefaction of Region I lies on the Γ- curve ab. Point b in Figure 2 is the image of OF in Figure 1, which consequently has a slope $dt/dx = -2/D$.

Reflection of the detonation wave from the free surface at $x = a$ (Figure 1) produces a rarefaction fan with straight C− characteristics centered at the point $P(x = a,\ t = a/D)$. The detonation gases are then accelerated into the void $(x > a)$ with limiting velocity D. Forward expansion is along af in Figure 2, and a region of overlap between the reflected rarefaction and the rarefaction following the detonation (the Taylor wave) is established as Region II of Figure 1.

For Region I:

$$\text{C}+:\quad u + c = x/t$$

$$\text{C}-:\quad u - c = -D/2$$

$$u = (x/t - D/2)/2 \tag{2a}$$

$$c = (x/t + D/2)/2 \tag{2b}$$

$$\rho = 8\rho_0(x/t + D/2)/9D \tag{2c}$$

Since Region I is a simple wave mapped on ab of Figure 2, any curve traversing Region I lies on ab. In particular the line PG, which separates Region II from Region I, lies on ab. PG is the leading C− characteristic of the fan passing through P of Figure 1, so

$$(dx/dt)_{PG} = u - c = (x - a)/(t - a/D)$$

But, since PG maps onto ab of Figure 2, $u - c = -D/2$ and

$$(x - a)/(t - a/D) = -D/2$$

PG is then parallel to OF, and for every other C− characteristic passing through P,

$$dx/dt = (x - a)/(t - a/D) > -D/2$$

For Region II:

$$\text{C}-:\quad u - c = (x - a)/(t - a/D)$$

$$\text{C}+:\quad u + c = x/t$$

$$u = [x/t + (x - a)/(t - a/D)]/2 \tag{3a}$$

$$c = [x/t - (x - a)/(t - a/D)]/2 \tag{3b}$$

$$\rho = 16\rho_0 c/9D \tag{3c}$$

Region II of Figure 1 maps into the triangle abf of Figure 2.

B. Case E2

The explosive is bounded at $x = 0$ by a rigid backing and at $x = a$ by a void. The flow field is shown in Figure 3. Region I behind the detonation front is a simple wave centered at (0, 0). Reflection at the free surface produces a backward-facing wave centered at A. The interaction of this wave with the Taylor wave and the rigid boundary produces the distinct and identifiable regions shown. Region III is a uniform state bounded by the last characteristic of the Taylor wave, OH, the leading characteristic of the reflection fan, AC, and the rigid boundary. The necessity for such a uniform region is shown in Figure 4. Here the point "a" is the Chapman-Jouget

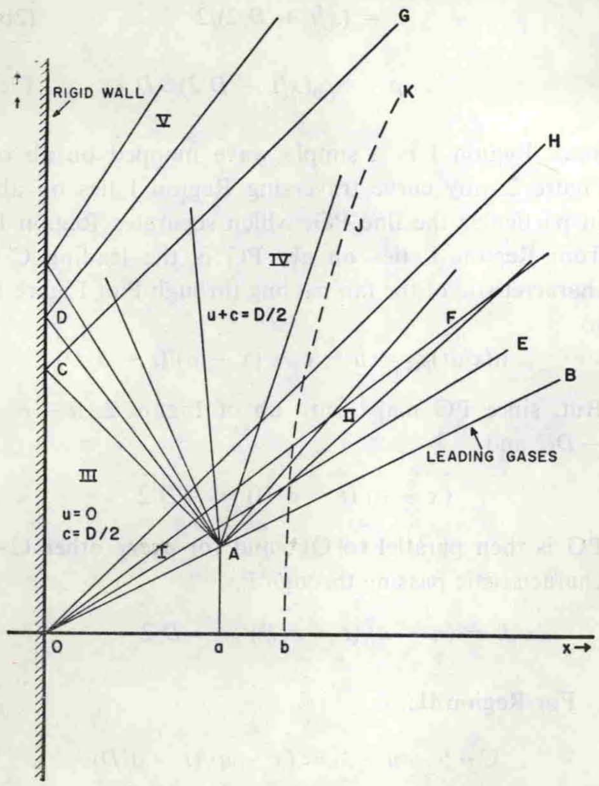

Figure 3

Flon field for confined explosive, case E2.

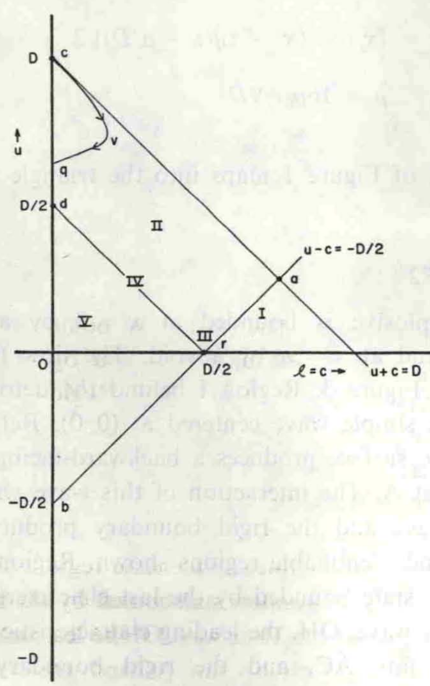

Figure 4

Hodograph mapping of flow field E1 of Figure 3.

state, and Region I lies along the Γ-characteristic ab. Region I terminates at $u = 0$, the condition imposed by the rigid boundary. Region III of Figure 3 is then mapped into the single point "III" of Figure 4. A traverse around the point A from OA to AB lies on or very close to the $\Gamma+$ characteristic ac in Figure 4. Region II of Figure 3 is mapped into the quadrilateral acdr of Figure 4. The boundary characteristic, OH, lies along dr. Region IV is again a simple wave region mapped onto the $\Gamma+$ characteristic $u + c = D/2$, shown in Figure 4. Region V is a mixed region resulting from interaction of the reflected rarefaction centered at A with its image in the $x = 0$ plane. The t-axis from C upward maps into the $u = 0$ axis, Or in Figure IV. The boundary CG corresponds to dr in Figure 4, and the open side of the triangular region V maps into Od of Figure 4. In symbols, these relations can be expressed as follows:

Region I: Same as Region I of Case E1.

Region II: Same as Region II of Case E1.

Region III:

$$u = 0 \tag{4a}$$

$$c = D/2 \tag{4b}$$

$$\rho = 8\rho_0/9. \tag{4c}$$

Region IV:

C+: $u + c = D/2$

C−: $u - c = (x - a)/(t - a/D)$

$$u = [D/2 + (x - a)/(t - a/D)]/2 \tag{5a}$$

$$c = [D/2 - (x - a)/(t - a/D)]/2 \tag{5b}$$

$$\rho = 16\rho_0 c/9D \tag{5c}$$

Region V:

C+: $u + c = (x + a)/(t - a/D)$

C−: $u - c = (x - a)/(t - a/D)$

$$u = x/(t - a/D) \tag{6a}$$

$$c = a/(t - a/D) \tag{6b}$$

$$\rho = 16\rho_0 a/9D(t - a/D) \tag{6c}$$